Overview of Grading for Induction Proofs

Recall the general structure for proofs by mathematical induction:

**Proof:** By mathematical induction

Let $P(n)$ be the statement: #1

**Basis:** $P(\#2)$ is true, because #3.

**Inductive step:** Suppose $k \geq \#2$. For the inductive hypothesis, suppose $P(k)$ is true: #4

We need to show that $P(k+1)$ is true:

#5.

**Heart of the inductive step: show $P(k+1)$ is true whenever $P(k)$ is**

Thus, for all $k \geq \#2$, the conditional $P(k) \rightarrow P(k+1)$ is true.

By the basis, inductive step, and the principle of mathematical induction, the claim is true.

Grading

**Disclaimer:** On any given problem, there may be small variations. However, in general, you should expect points on a 15-point problem to be assigned roughly in the following way:

**General format:** (2-3 points)

- Is there an explicit statement of the claim to be proved?
- Are the start of the proof and the method of proof (e.g., mathematical induction or structural induction) explicitly labeled?
- Is there an explicit and correct statement of the generic property $P$ (e.g., “Let $P(n)$ be the statement ...”)?
- Is there an explicit and appropriate wrap-up statement at the end of the proof (“By the basis, inductive step, and ...”).

**Basis step:** (3-4 points)

- Is the basis step clearly labeled?
- Does the basis step handle the correct case(s)?
- Is there an explicit statement that $P$ holds for the basis cases?
- Is there a clear, explicit, and correct verification that $P$ holds for all basis cases?
**Inductive step:** (8-10 points)

- Is the inductive step clearly labeled?
- Are necessary assumptions *besides the inductive hypothesis* spelled out (e.g., “Suppose $k \geq b$”)?
- Is the inductive hypothesis (e.g., $P(k)$) clearly labeled and spelled out?
- Is what need to be proved (e.g., $P(k + 1)$) clearly labeled and spelled out?
- Is the heart of the inductive step clear and correct?
  *This part is usually worth 4-6 points, but it will be looked at only if the inductive hypothesis and what’s needed to be proved are correct.*
- Is there an explicit and appropriate wrap-up statement at the end of the inductive step (e.g., “Thus, for all $k \geq b$, the conditional $P(k) \rightarrow P(k + 1)$ is true”)?