Propositional Logic: What We Didn’t Finish Last Time

Prof. Susan Older

6 September 2016
**Logical Consequence: A Generalization**

**Definition**

Suppose $A_1, \ldots, A_n$ and $B$ are propositional formulas.

\[ B \text{ is a logical consequence of } A_1, \ldots, A_n \text{ provided that } B \text{ is a logical consequence of the conjunction } A_1 \land A_2 \land \cdots \land A_n. \]
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Is $B$ a logical consequence of $A_1, \ldots, A_n$?
To find out:

1. Find all rows in truth table in which $A_1, \ldots, A_n$ are all true.
2. Look at the values of $B$ only in the rows you found in Step 1.
   - If any of those values of $B$ are false, the answer is No.
   - Otherwise, the answer is Yes.
3. If you found no rows in Step 1, the answer is still Yes.
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If you found no rows in Step 1, the answer is still Yes.
Logical Consequence: Example #1

Example

Is $C$ a logical consequence of $H_1$, $H_2$, and $H_3$?

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$C$ is not a logical consequence of $H_1$, $H_2$, and $H_3$. 
Logical Consequence: Example #2

Example

Is $D$ a logical consequence of $H_1$, $H_2$, and $H_3$?

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Logical Consequence: Example #2

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Is \( D \) a logical consequence of \( H_1, H_2, \) and \( H_3 \)?

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$D$ is a logical consequence of $H_1$, $H_2$, and $H_3$. 
Is $D$ a logical consequence of $H_1$, $H_2$, $H_3$, and $H_4$?

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There are no rows where $H_1$, $H_2$, $H_3$, and $H_4$ are all true. Therefore, $D$ is a logical consequence of $H_1$, $H_2$, $H_3$, and $H_4$. 

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6 September 2016
Logical Consequence: Example #3

Example

Is \( D \) a logical consequence of \( H_1, H_2, H_3, \) and \( H_4 \)?

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There are no rows where \( H_1, H_2, H_3, \) and \( H_4 \) are all true.
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Is $D$ a logical consequence of $H_1$, $H_2$, $H_3$, and $H_4$?

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To English/MathSpeak and Back Again

\( p \wedge q \)

- “\( p \) and \( q \)”
- “\( p \) but \( q \)”
To English/MathSpeak and Back Again

$p \land q$
- “p and q”
- “p but q”

$p \lor q$
- “either p or q (or both)”
- “at least one of p and q is true”
To English/MathSpeak and Back Again

\( p \land q \)
- “p and q”
- “p but q”

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- “either p or q (or both)”
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\( \neg(p \lor q) \equiv \neg p \land \neg q \)
- “neither p nor q”
- “p and q are both false”
- “it is not the case p and it is not the case q”
To English/MathSpeak and Back Again

\( p \land q \)
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\( \neg (p \land q) \equiv \neg p \lor \neg q \)
- “at least one of p and q is false”
- “it is not the case that both p and q are true”
Conditional Love: More English/MathSpeak

\[ p \rightarrow q \]

- “If \( p \), then \( q \).”
- “\( p \) only if \( q \)”
- “not \( p \) unless \( q \)”
- “Whenever \( p \), it follows that \( q \)”
- “\( p \) is a sufficient condition for \( q \)”
- “\( q \) is a necessary condition for \( p \)”

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Conditional Love: More English/MathSpeak

\[ p \rightarrow q \]

- “If p, then q.”
- “p only if q”
- “not p unless q”
- “Whenever p, it follows that q”
- “p is a sufficient condition for q”
- “q is a necessary condition for p”

\[ p \leftrightarrow q \]

- “p if and only if q” (also: “p iff q”)
- “p is necessary and sufficient for q”
- “p is true exactly when q is true”
Let’s Practice!

Consider the following propositions:

\[ p : \text{the message is scanned for viruses} \]
\[ q : \text{the message was sent from an unknown system} \]

Translate these sentences into propositional logic:

1. The message is scanned for viruses whenever the message was sent from an unknown system.
2. The message was sent from an unknown system, but it was not scanned for viruses.
3. It is necessary to scan the message for viruses whenever it was sent from an unknown system.
4. When a message is not sent from an unknown system, it is not scanned for viruses.

Let’s Practice!

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    p : \text{the message is scanned for viruses} \\
    q : \text{the message was sent from an unknown system}
\]

Translate these sentences into propositional logic:

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- \( p \) : the message is scanned for viruses
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Translate these sentences into propositional logic:

1. The message is scanned for viruses whenever the message was sent from an unknown system. \( q \rightarrow p \)
2. The message was sent from an unknown system, but it was not scanned for viruses. \( q \wedge \neg p \)
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2. The message was sent from an unknown system, but it was not scanned for viruses. \( q \land \neg p \)
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Let’s Practice!

Consider the following propositions:

- \( p \): the message is scanned for viruses
- \( q \): the message was sent from an unknown system

Translate these sentences into propositional logic:

1. The message is scanned for viruses whenever the message was sent from an unknown system. \( q \rightarrow p \)
2. The message was sent from an unknown system, but it was not scanned for viruses. \( q \land \neg p \)
3. It is necessary to scan the message for viruses whenever it was sent from an unknown system. \( q \rightarrow p \)
4. When a message is not sent from an unknown system, it is not scanned for viruses. \( \neg q \rightarrow \neg p \)