ASSIGNMENT 1

DUE BY: 08:25 am on Thursday, the 5th of February, 2004, at 125 Link Hall.

[No credit for late assignments. No assignments will be accepted in class on the due date.]

[Please feel free to consult with me; a strong correlation with a classmate’s solution may force me to take action.]

{Total points = 100}

1. (a) Let \(x_1[n]\) and \(x_2[n]\) be periodic sequences with fundamental periods \(N_1\) and \(N_2\), respectively. Under what conditions is the sum \(x[n] = x_1[n] + x_2[n]\) periodic, and what is the fundamental period if it is periodic? \(5\) points

(b) If \(x_1[n] = \cos(0.2\pi n)\) and \(x_2[n] = \cos(0.125\pi n)\), determine if \(x_1[n]\) is periodic and determine if \(x_2[n]\) is periodic. If so, determine the number of samples per fundamental period for each sequence. \(5\) points

(c) Determine if the sum of \(x_1[n] + x_2[n]\) is periodic. If so, determine the number of samples per fundamental period. \(5\) points

2. Determine (i) the minimum number of bits, (ii) the minimum sampling rate, and (iii) the bit rate that you would require to digitally capture information from the following analog sources (please state your assumptions or references clearly)
   - AM radio \(5\) points
   - FM radio \(5\) points
   - Plain old telephone \(5\) points
   - 6-channel live recording Tchaikovsky’s 1812 Overture \(5\) points


4. Problem 9.18(a)(b) from Phillips, Parr, and Riskin (third edition) \(5\) points

5. Problem 9.21(a) from Phillips, Parr, and Riskin (third edition) \(5\) points

6. Problem 9.22(a)(b)(c) from Phillips, Parr, and Riskin (third edition) \(15\) points

7. Problem 9.25(a)(b) from Phillips, Parr, and Riskin (third edition) \(10\) points

8. Problem 9.28 from Phillips, Parr, and Riskin (third edition) \(5\) points

9. Prove the following equalities:

   (a) \[\sum_{k=0}^{N} \alpha^k = \frac{1-\alpha^{N+1}}{1-\alpha}, \text{ for } \alpha \neq 1 \text{ OR } (N+1) \text{ for } \alpha = 1\] \(5\) points

   (b) \[\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \text{ for } |\alpha|<1\] \(5\) points

   (c) \[\sum_{k=0}^{\infty} \alpha^k = \frac{\alpha^m}{1-\alpha}, \text{ for } |\alpha|<1\] \(5\) points

   (d) \[\sum_{k=0}^{\infty} k \alpha^k = \frac{\alpha}{(1-\alpha)^2}, \text{ for } |\alpha|<1\] \(5\) points