RECONSTRUCTION OF THE NON-MINIMUM PHASE FUNCTION FROM AMPLITUDE ONLY DATA

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ABSTRACT

A procedure for non-minimum phase reconstruction from magnitude only data has been presented in this paper. Utilizing the fact that the auto-correlation of the time domain sequence is equal to the discrete cosine transformation of the square of the magnitude response, non-minimum phase component can be recovered. The non-minimum phase reconstruction does not have a unique solution. We select that solution which has a causal time domain response. The theory has been applied to extract the phase of the scattering parameters of microwave filters to illustrate the applicability of this approach.

INTRODUCTION

The phase reconstruction from magnitude only data is not a unique problem. If one assume that the signal is causal, i.e. $f(t)=0$ for t<0, the minimum phase can be reconstructed by the complex cepstrum analysis [1]. The minimum phase refers that the zeros of the transfer function lie in the left half of the s-plane ($s=\sigma+j\omega$ with $\sigma<0$) or inside the unit circle in the z-plane ($z=e^{js}$ with $|z|<1$). That is the minimum phase system is causal and forms a stable system and the system has a causal and stable inverse. It can be easily shown that the log magnitude and the phase sequence have Hilbert transformation relationships. If $a[n]$ is a time domain sequence, the complex cepstrum of $a[n]=a_n$ is defined as

$$\hat{A}(z)=\log[A(z)], \qquad (1)$$

where $A(z)=\sum_{n=-\infty}^{\infty}a_n z^{-n}$ and $\hat{A}(z)=\sum_{n=-\infty}^{\infty}\hat{a}_n z^{-n}$.

Since causal sequences have a region of convergence of the form $r_R<|z|$, there is no singularity of $\log[A(z)]$ on or outside the unit circle if $\hat{a}_n$ is causal and stable.

$$\hat{A}(z)=\sum_{n=-\infty}^{\infty}\hat{a}_n z^{-n}$$

$$=\log[A(z)]=\log\left[|A(z)|e^{j\angle A(z)}\right]=\log|A(z)|+j\angle A(z),$$

(2)

where $j^2=-1$ and $\angle(.)$ is a phase component.

Since $\hat{a}_n$ is causal,

$$\mathrm{H}[\log|A(z)|]=\angle A(z) \qquad (3)$$

where H[.] represent the Hilbert transformation. Therefore the minimum phase can be obtained by taking a Hilbert transformation of log magnitude of the sequence. If the system is not minimum phase (i.e. when some of the zeros of the transfer function may be on the right half plane) then (3) does not hold. Most electromagnetic system has a non-minimum phase response and the cepstrum approach given by (3) cannot be used for the practical non-minimum phase realization. The only constraint to get the non-minimum phase response is to force causality on the system. The linear difference in phase is inevitable because of the linear phase difference is equivalent to the pure delay in the time domain. Since we are dealing with linear shift invariant system changing the impulse response of the system by a time shift does not change the transfer function of the original system except that the phase is modified by a linear function. The slope of the linear difference is corresponding to the time delay.

FORMULATION

Assume a causal sequence a_n where n=1,2,…,N then its spectrum is

$$A(\omega)=\sum_{n=0}^{\infty}a_n e^{-j\omega n+\Phi_n}=R(\omega)+jI(\omega), \qquad (4)$$

where Φ_n is certain phase shifts associated with

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a_n. The magnitude response of the system, i.e., power spectrum, is

$$|A(\omega)|^2 = |R(\omega)+jI(\omega)|^2$$
$$=|R(\omega)|^2 + |I(\omega)|^2$$
$$=\left|\sum_{n=0}^{\infty} a_n \cos(n\omega+\Phi_n)\right|^2 + \left|\sum_{n=0}^{\infty} a_n \sin(n\omega+\Phi_n)\right|^2 . \quad (5)$$

Assumming a_n as a real sequence will yield

$$=[a_1\cos(\omega+\Phi_1)+a_2\cos(2\omega+\Phi_2)+\cdots]^2$$
$$+[a_1\sin(\omega+\Phi_1)+a_2\sin(2\omega+\Phi_2)+\cdots]^2$$
$$=[a_1{}^2\cos^2(\omega+\Phi_1)+a_2{}^2\cos^2(2\omega+\Phi_2)+\cdots+2a_1a_2\cos(\omega+\Phi_1)\ \cos(2\omega+\Phi_2)+\cdots]$$
$$+[a_1{}^2\sin^2(\omega+\Phi_1)+a_2{}^2\sin^2(2\omega+\Phi_2)+\cdots+2a_1a_2\sin(\omega+\Phi_1)\ \sin(2\omega+\Phi_2)+\cdots]$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_n a_m \cos[(n-m)\omega+\Phi_m-\Phi_n]. \quad (6)$$

Multiply both side with $\cos[k\omega+\Phi_m-\Phi_n]$, where k is an positive integer, and integrte from 0 to 2π. Then,

$$\int_0^{2\pi}\cos(k\omega)\ |A(\omega)|^2 d\omega$$
$$=\int_0^{2\pi}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_n a_m \cos[(m-n)\omega+\Phi_m-\Phi_n]\cos(k\omega)\,d\omega$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_n a_m \int_0^{2\pi}\cos[(m-n)\omega+\Phi_m-\Phi_n]\cos(k\omega)\,d\omega$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_n a_m \int_0^{2\pi}\begin{Bmatrix}\cos[(m-n)\omega]\cos(\Phi_m-\Phi_n)\\ -\sin[(m-n)\omega]\sin(\Phi_m-\Phi_n)\end{Bmatrix}\cos(k\omega)\,d\omega$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_n a_m \int_0^{2\pi}\begin{Bmatrix}\cos(k\omega)\cos[(m-n)\omega]\cos(\Phi_m-\Phi_n)-\\ \cos(k\omega)\sin[(m-n)\omega]\sin(\Phi_m-\Phi_n)\end{Bmatrix}d\omega$$
. (7)

The last term exist only when $|m-n|=k$. When $|m-n|=k$, the integral is evaluated to be π. That is,

$$\int_0^{2\pi}\cos(k\omega)\ |A(\omega)|^2 d\omega$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_n a_m \int_0^{2\pi}\cos(k\omega)\cos[(m-n)\omega]\cos(\Phi_m-\Phi_n)\,d\omega$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_n a_m \pi \quad (8)$$

or equivently,

$$\int_0^{2\pi}\cos(k\omega)\ |A(\omega)|^2 d\omega = 2\pi\sum_{n=0}^{\infty} a_n a_{n+k} . \quad (9)$$

The left hand side of (9) is the cosine transformation of the magnitude response and the right hand side is the autocorrelation of the time domain sequence. Since we are dealing with finite number of data, N data, (9) will turn to a discrete from

$$\Delta\omega\sum_{i=0}^{N}\cos(k\omega_i)\ |A(\omega_i)|^2 = 2\pi\sum_{n=0}^{N} a_n a_{n+k} . \quad (10)$$

Left hand side of (10) is symmetric with respect to $\omega=\pi$ if $A(\omega)$ is symmetric then this portion corresponds to the negative index of n in (10). If a_n is not a causal sequence, that is the right hand side of (10) becomes asymmetric then the equation would not hold. Therefore, the time domain sequence a_n need to be causal for equation (10) to hold.

Note that the auto-correlation in (10) will yield the same result for a delayed version of a_n as long as the delayed version of a_n is causal and thus it does not have a unique solution. The nonuniqueness results in a linear phase response with respect to the angular frequency. One can utilize an optimization to solve for the coefficient a_n to minimize the error function $Er(a_n)$ given by

$$Er(a_0,a_1,\cdots,a_{N-1}) = \left|\int_0^{2\pi}\cos(k\omega)|A(\omega)|^2 d\omega - 2\pi\sum_{n=0}^{\infty} a_n a_{n+k}\right|$$
. (11)

Since (11) represent a 2N-dimensional search over a_0 through a_{N-1} where a_n is a complex number in general, the computational load can be very large. Also the gradient with respect to each a_n is not easy to compute. To avoid this situation an all-pass filter representation of $A(\omega)$ has been used. That is,

$$A(\omega) = A_{\min}(\omega)\cdot H_{all}(\omega), \quad (12)$$

where $A_{\min}(\omega)$ is a minimum phase form of $A(\omega)$. From (3)

$$A_{\min}(\omega) = |A(\omega)|e^{jH(\log|A(\omega)|)} \quad (13)$$

and $H_{all}(\omega)$ is the impulse response of an all-pass filter [1].

$$H_{all}(\omega)=\prod_{m=0}^{P-1}\frac{e^{-j\omega}-d_m}{1-d_m e^{-j\omega}}\cdot\prod_{n=0}^{Q-1}\frac{\left(e^{-j\omega}-b_n^*\right)\left(e^{-j\omega}-b_n\right)}{\left(1-b_n^* e^{-j\omega}\right)\left(1-b_n e^{-j\omega}\right)}$$

, (14)

where d_m is a real pole-zero of the all-pass filter, b_n is a complex pole-zero of the all-pass filter, and b_n^* is a complex conjugate.

An error function which utilizing all-pass filter will be expressed as

$$Er_{all}(d_0,d_1,\cdots,d_{P-1},b_0,b_1,\cdots,b_{Q-1})$$

$$=\left|\int_0^{2\pi}\cos(k\omega)|A(\omega)|^2 d\omega-2\pi\cdot Corr\{\Im^{-1}[A_{\min}(\omega)\cdot H_{all}(\omega)]\}\right|$$

, (15)

where $Corr(a_n)=\sum_{n=0}^{N-1}a_n a_{n+k}$, $\Im^{-1}(\cdot)$ is an inverse Fourier transformation and $H_{all}(\omega)$ is given by (14).

By this change of optimization variables, 2N dimensional search turns to a 2P+2Q dimensional search. As will be shown in the numerical example, a_n are significant for the first few values and almost zero for the rest of the sequence. From the first numerical example shown in figure 2(b), it turns out that in (11) we need 20 to 30 dimensional search to obtain the minimum error since the inverse Fourier transform of $A(\omega)$ decays to zero for after 30 coefficients. Note that the real part of $A(\omega)$ is symmetric and a_n is a real number. (15) needs only 3-6 dimensional search for minimization though it requires more complicated computation than (11). Hence, this is a great computational improvement.

NUMERICAL EXAMPLE

The response of a comb line filter was computed as an example and the structure is given in figure 1. Using the computer program LINPAR [4] and MATPAR [5], the scattering parameter could be obtained. The pass-band corresponds from 420MHz to 520MHz and the magnitude response is shown in figure 2(a). The image (horizontal argument from 1 to 300) was taken as the mirror image to have symmetry so that real valued time domain coefficients are realized (Note that the Fourier transformation of real sequence is conjugate-symmetric and the magnitude is symmetric).

Figure 1. Structure of a comb line filter

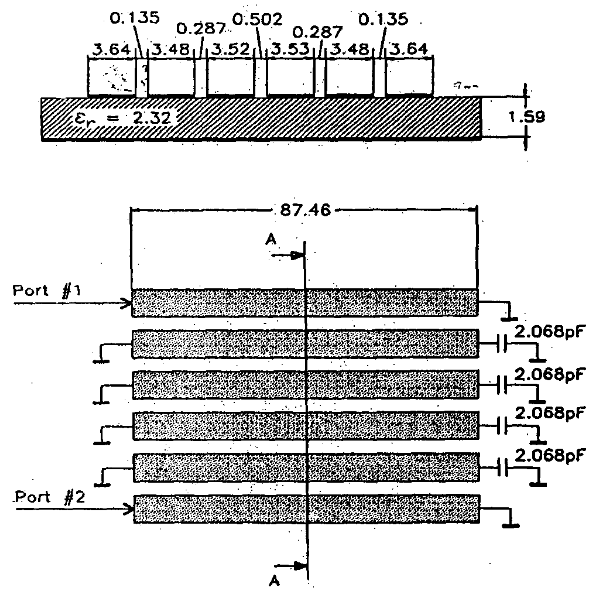


Figure 2. Response of the comb line filter.

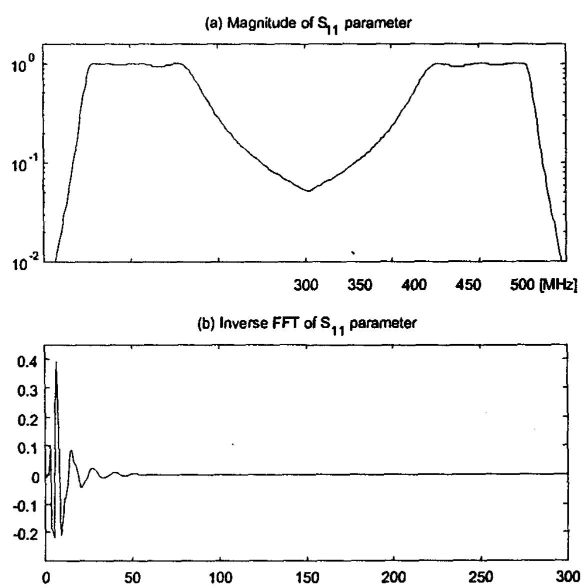


Figure 3. Phase response of original, the minimum phase and the reconstructed response of the example.

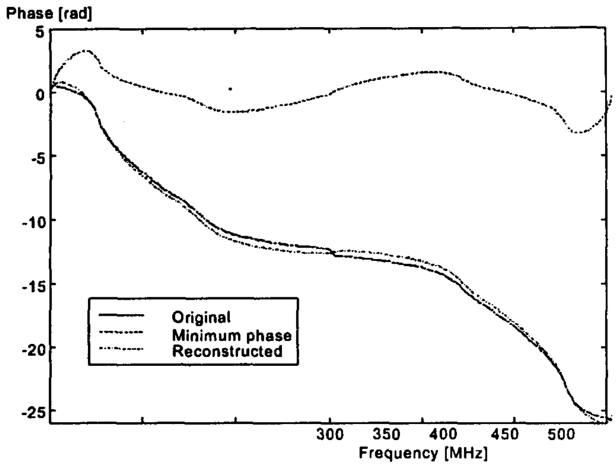
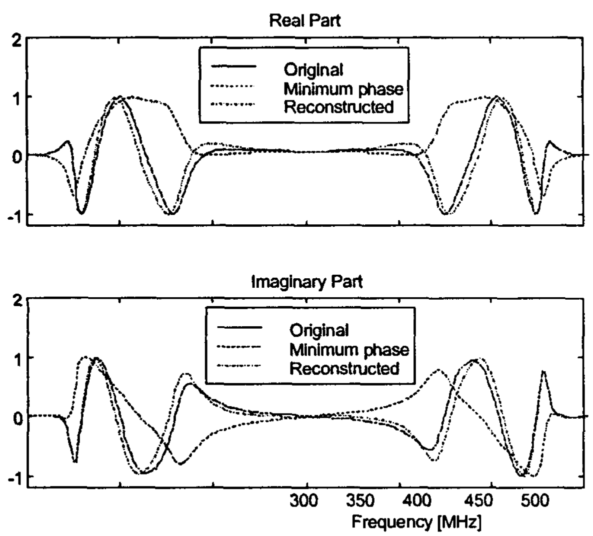


Figure 4. Original s parameter from MATPAR, the estimated and the minimum phase from equation (3) are compared.



P=0 and Q=3 was chosen in (14) which means that only three pole-zeros were optimized. Our simulation shows that increasing poles and zeros returned $d_m = -1$ and $b_n = 1$ which does not enhance the optimization result due to cancellation of the numerator and the denominator.

The reconstructed phase response by minimizing (15) was plotted in figure 3. Since the time domain coefficient starts almost from the origin in time, there is not much time delay between the original sequence and the optimized sequence and there are no large linear differences in the phase reconstruction. Figure 4 shows the comparison of the real and imaginary part of the s parameter of the comb line filter. The plot has the original, the reconstructed and the minimum phase response.

CONCLUSIONS

A method utilizing auto-correlation and discrete cosine transformation to reconstruct the non-minimum phase component from the magnitude only data has been presented in this paper. Optimization by pole-zero pairs using an all-pass filter have been done to solve this problem. In the numerical example, the phase of micro strip line filter have been reconstructed. Taking more pole-zero pairs for optimizations may provide better results but it takes more computation time.

ACKNOWLEDGEMENT

Grateful acknowledgement is made to Prof. Dante Youla of Polytechnique University of Brooklyn for suggesting the improved computational methodology.

REFERENCES

1. A. V. Oppenheim and R. W. Schafer, "Discrete-time signal processing", Prentice Hall, Englewood Cliff, New Jersey, 1989.
2. M. H. Hayes, "The reconstruction of a multidimensional sequence from the phase or magnitude of its Fourier transform", IEEE Trans. ASSP, vol. ASSP-30, No. 2, April 1982, pp. 140-154.
3. A. R. Djordjevic et.al., "Analysis of wire antennas and scatterers", Artech House, Boston, 1990.
4. A. R. Djordjevic et.al., "Matrix parameters for multiconductor transmission lines", Artech House, Norwood, MA, 1989.
5. A. R. Djordjevic et.al., "Scattering parameters of microwave networks with multiconductor transmission lines: software and user's manual", Artech House, Boston, 1990.
6. W. L. Stulzman and G. A. Thiele, "Antenna theory and design", John Wiley & Sons, New York, 1981.