Problem 13: Let $f: A \to B$ such that $f(g(x)) = x$, all $x \in B$.
Let $g(B) = \{g(x) \mid x \in B\}$. $g(B) \subseteq A$. $\|g(B)\| = \|B\| = n$, since g is 1-1.
f maps $A - g(B)$ to B. The size of the set of functions $f: A \to B$ such that $f(g(x)) = x$ is the size of the set of functions $(A - g(B)) \to B$, which is $\|B\|^{\|A - g(B)\|} = n^{m-n}$.

Problem 15: Suppose $S = D \times E$. Then $(0,0) \in D \times E$ and $(1,1) \in D \times E$. Therefore $0 \in D$ and $1 \in E$. So, $(0,1) \in D \times E$. $\therefore (0,1) \in S$. But $(0,1) \notin S$. Contradiction. Therefore there are no sets D, E such that $S = D \times E$.

Problem 17:

$$a \xrightarrow{R} b \xrightarrow{S} c \xrightarrow{R} d \xrightarrow{S} e.$$

R, S are transitive. $R \circ S$ is not transitive.

Problem 20: If $x \in \omega'$ is finite, then $\cup x = \{0, 1, \ldots, x\}$.
If $x = \omega$, then $\cup\{0, 1, \ldots\} = \{0, 1, \ldots\} = \omega$. $\omega \notin \cup\omega$, but $\cup\cup\omega = \cup\omega = \omega$.

Problem 21: Consider the following sets:
T_0^n = trit-strings of length n ending in 0.
T_1^n = trit-strings of length n ending in 1.
T_2^n = trit-strings of length n ending in 2.
T^n = trit-strings of length n.

$\|T^n\| = \|T_0^n\| + \|T_1^n\| + \|T_2^n\|$.

$\|T_0^{n+1}\| = \|T_1^n\| + \|T_2^n\|$

$$\begin{aligned}\|T_1^{n+1}\| = \|T_2^{n+1}\| = \|T^n\| &= \|T_0^n\| + 2\|T_1^n\| \\ &= 2\|T_1^{n-1}\| + 2\|T^{n-1}\| \\ &= 2\|T^{n-2}\| + 2\|T^{n-1}\| \\ &= 2(\|T^{n-2}\| + \|T^{n-1}\|).\end{aligned}$$

$\|T^0\| = 1$; $\|T^1\| = 3$; $\|T^2\| = 8$; $\|T^3\| = 22$; $\|T^4\| = 60$; $\|T^5\| = 164$; $\|T^6\| = 328$