# Review Questions

# CSE/CIS 607
# Fall, 2007

This practice exam is 3 to 4 times longer than the actual final exam. Also, question 17 itself is almost the length of the actual final exam. The questions are intended to give you review *practice*.

**Question 1:** Assign polarity to each of the subformula occurrences in the following formula

$$(\neg P \supset ((Q \equiv P) \supset Q)) \vee (P \wedge \neg R)$$

**Question 2:** Let $\Psi$ be a formula in which the subformula $P \vee \neg P$ has exactly one occurrence, and that occurrence has negative polarity <u>in $\Psi$</u>. Is the following formula valid? If it is valid, explain why. If it is not valid give a (simple) counterexample.

$$(\Psi[P \vee \neg P] \wedge Q) \supset (\Psi[P \wedge \neg P] \wedge Q)$$

**Question 3:** Intentionally omitted.

**Question 4:** Use tableau methods to prove that the following formula is valid:

$$\exists x[p(x) \wedge \forall y[p(y) \supset q(x,y)]] \supset \exists x[p(x) \wedge q(x,x)]$$

**Question 5:** Is the following a valid formula?

$$(\forall x[p(x) \supset q(x)] \wedge \exists x[q(x) \wedge r(x)]) \supset \forall x[p(x) \supset r(x)]$$

If it is, prove it (a tableau-style proof would do). If it is not valid, give an interpretation $\mathcal{A}$ in which the formula is false.

**Question 6:** Find a most general unifier of the two terms

$$g(g(w, g(b, z)), w) \quad \text{and} \quad g(g(y, g(b, u)), g(u, z))\,.$$

**Question 7:** Prove that the following tableau is valid

**a1.** $p(g(y, u)) \vee q(g(x, f(x))) \vee p(g(x, f(x)))$
**a2.** $p(g(x, f(x))) \vee \neg q(g(y, u))$
**g1.** $p(g(y, u)) \vee s(g(x, f(x)))$

**Question 8:** Find a prenex normal form equivalent to

$$\exists x[p(x, y) \supset \exists x[q(x) \vee \neg \exists y r(y)]]$$

**Question 9:** Skolemize as an assertion:

$$\forall x \exists y \forall z[(p(x) \supset q(x)) \wedge ((q(y) \wedge r(y)) \supset (p(z) \supset r(z)))]$$

**Question 10:** Skolemize as a goal:

$$\forall x \exists y \forall z[(p(x) \supset q(x)) \wedge ((q(y) \wedge r(y)) \supset (p(z) \supset r(z)))]$$

**Question 11:** Suppose

$$\mathcal{A} \models \forall x[r(x,x)]$$
$$\mathcal{A} \models \forall x \forall y[r(f(x),f(y)) \supset r(x,y)]$$
$$\mathcal{A} \models \exists x \forall y[\neg r(x,f(y))]$$

Prove that $|\mathcal{A}|$ is an infinite set.

**Question 12:** Use tableau methods to prove that following tableau is valid:

a1. $\forall x[P(x) \supset (Q(x) \wedge R(x))]$
a2. $\forall x[Q(x) \supset S(x)]$
g1.$\exists x P(x) \supset \exists x S(x)$

**Question 13: Question 2:** Find a single interpretation $\mathcal{A}$ in which the formula

$$\forall x P(x) \supset Q$$

is true, and the formula

$$\forall x[P(x) \supset Q]$$

is false.

**Question 14:** Find a most general unifier of the two terms

$$g(x,y,f(x),z,u) \quad \text{and} \quad g(a,x,z,f(x),h(v,z))\,.$$

**Question 15:** Find a formula in prenex form equivalent to the following and Skolemize it as an assertion.

$$\exists x P(f(x)) \wedge \forall x[Q(a) \supset P(g(x,a))]$$

**Question 16:** Find a formula in prenex form equivalent to the following and Skolemize it as a goal.

$$\exists x P(f(x)) \wedge \forall x[Q(a) \supset P(g(x,a))]$$

**Question 17:** Prove, using tableau, the following

$$\forall x \, \forall y \, \forall z \, [x + (y + z) = (x + y) + z]$$

in Peano Arithmetic (PA).
**Hint:** Do induction on $z$. You may used previously proved theorems of PA that we proved in the course. [WARNING: This is long. It is intended to give you practice doing induction proofs. The induction step itself involves an induction on $y$.]

**Question 18:** Let $A$ be the set of natural numbers recursively defined by

1. $(0, 0) \in A$.
2. If $(m, n) \in A$, then $(m + 2, n + 3) \in A$ and $(m + 3, n + 2) \in A$.

Use structural induction to prove that if $(m, n) \in A$ then 5 divides $m + n$.
[You do not need to give a tableau proof, but you do need to give a rigorous proof.]