

CIS 607: Mathematical Basis for Computing

Exam 1 Solutions

Daniel R. Patten
 Department of Electrical Engineering and Computer Science
 Syracuse University
 Syracuse, NY 13244

October 13, 2009

Problem 1. $((P^+ \supset^- Q^-) \supset^+ (Q^\pm \equiv^+ (P^\pm \vee^\pm Q^\pm))) \vee^+ (\neg^- R^+ \supset^+ R^+)$

Problem 2. Consider the tableau below.

a1.	$\neg P$	given
a2.	$\neg P$	given

It is invalid since its associated sentence P is not necessarily true. However, by using the proposed rule, we obtain a valid tableau.

a1.	$\neg P$	given
a2.	$\neg P$	given
a3.	$\neg \text{false} \supset \neg \text{true}$	a1, a2, proposed rule
a4.	false	a3, rew

Since the proposed rule does not preserve validity, it is not valid.

Problem 3.

(1)

a1.	$P \supset \neg E$
a2.	$\neg E \supset (\neg B \vee \neg P)$
a3.	$P \supset (B \equiv \neg E)$
g1.	$\neg P$

(2) Suppose that the goal does not follow from the assertions. This occurs if and only if the assertions are true but the goal is false. Thus, by g1 it follows that P is true, and so by a1, the sentence E is false, whence by a3, the sentence B is true. By a2, we infer that $\neg B \vee \neg P$ is true, but

since B is false, this implies that $\neg P$ is true, in contradiction to previous result that P is true. Therefore, we conclude that the goal follows from the assertions.

Problem 4.

a1.	$P \supset (Q \vee R)$	given
a2.	$P \supset (\neg Q \wedge R)$	given
g1.	$\neg P \vee R$	given
g2.	$\neg P$	g1, \vee -split
g3.	R	g1, \vee -split
g4.	$\neg(P \supset (Q \vee \text{false})) \wedge \text{true}$	a1, g3, AG-res
g5.	$P \wedge \neg Q$	g4, rew
g6.	$\neg \text{false} \wedge (\text{true} \wedge \neg Q)$	g2, g5, GG-res
g7.	$\neg Q$	g6, rew
g8.	$\neg(P \supset (\neg \text{true} \wedge S)) \wedge \neg \text{false}$	g7, a2, GA-res
g9.	P	g8, rew
g10.	$\text{true} \wedge \neg \text{false}$	g2, g9, GG-res
g11.	true	g10, rew

Problem 5. The following chain of equivalences establishes the desired result.

$$\begin{aligned}
x \equiv y &\equiv (x \supset y) \wedge (y \supset x) \\
&\equiv \neg(x \wedge \neg y) \wedge \neg(y \wedge \neg x) \\
&\equiv \neg(x \wedge (y|y)) \wedge \neg(y \wedge (x|x)) \\
&\equiv (x|(y|y)) \wedge (y|(x|x)) \\
&\equiv \neg((x|(y|y)) \wedge (y|(x|x))) \\
&\equiv \neg((x|(y|y))|(y|(x|x))) \\
&\equiv ((x|(y|y))|(y|(x|x))|(x|(y|y))|(y|(x|x)))
\end{aligned}$$